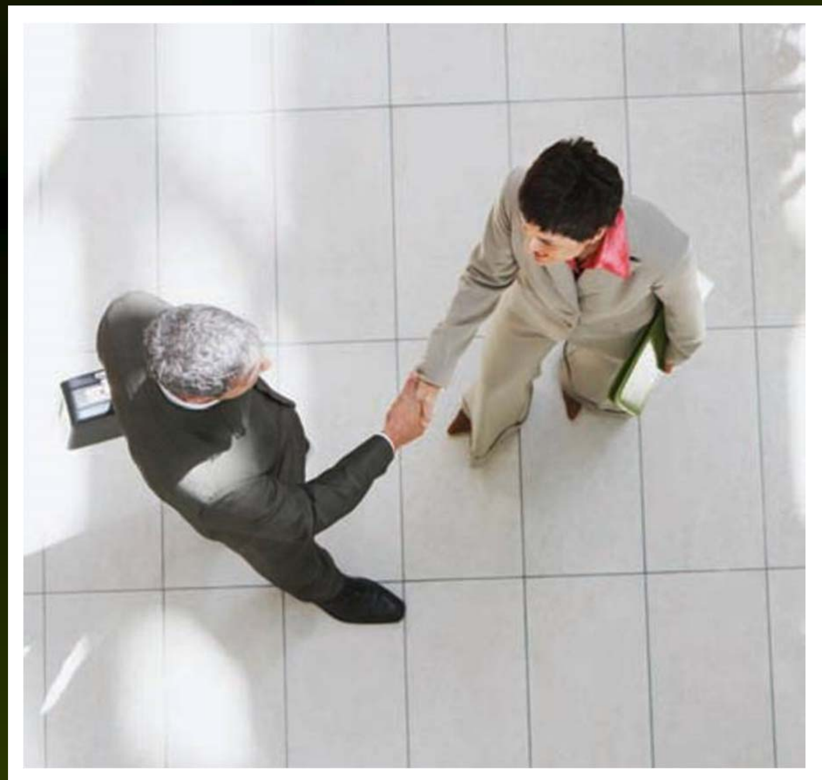
1 PRELIMINARIES



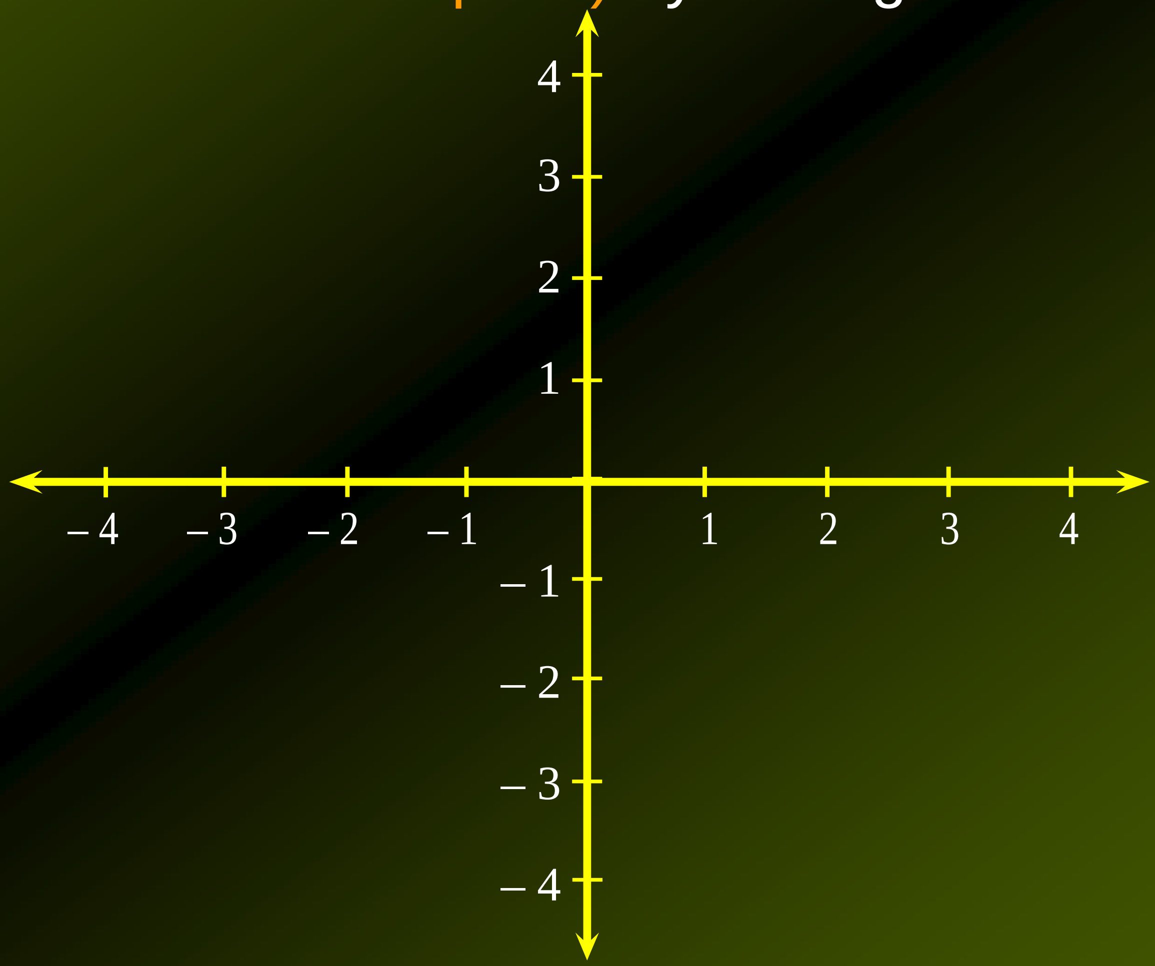
1.3 The Cartesian Coordinate System

The Cartesian Coordinate System

At the beginning of the chapter we saw a one-to-one correspondence between the set of real numbers and the points on a straight line (one dimensional space).

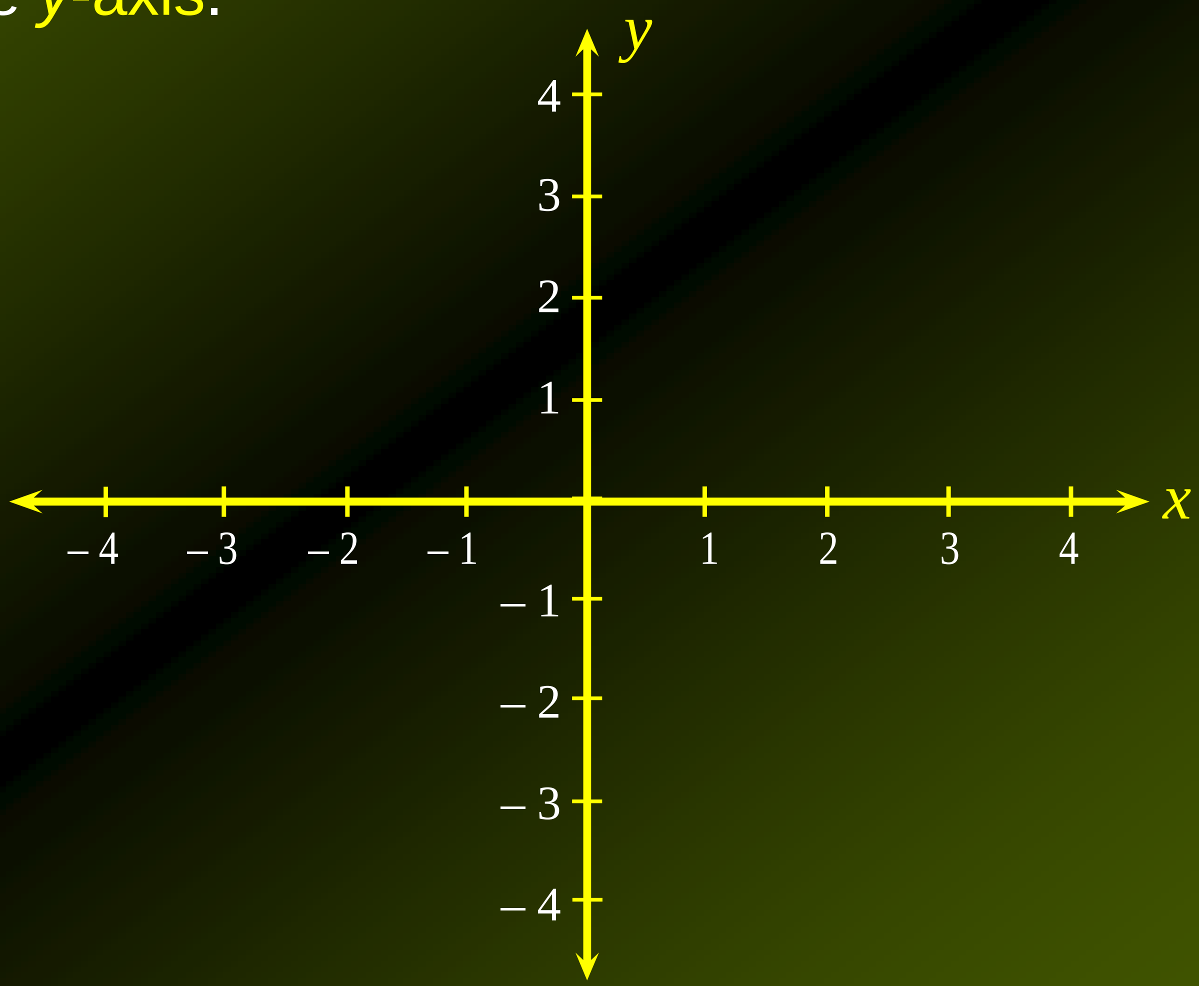
The Cartesian Coordinate System

The Cartesian coordinate system extends this concept to a plane (two dimensional space) by adding a *vertical axis*.



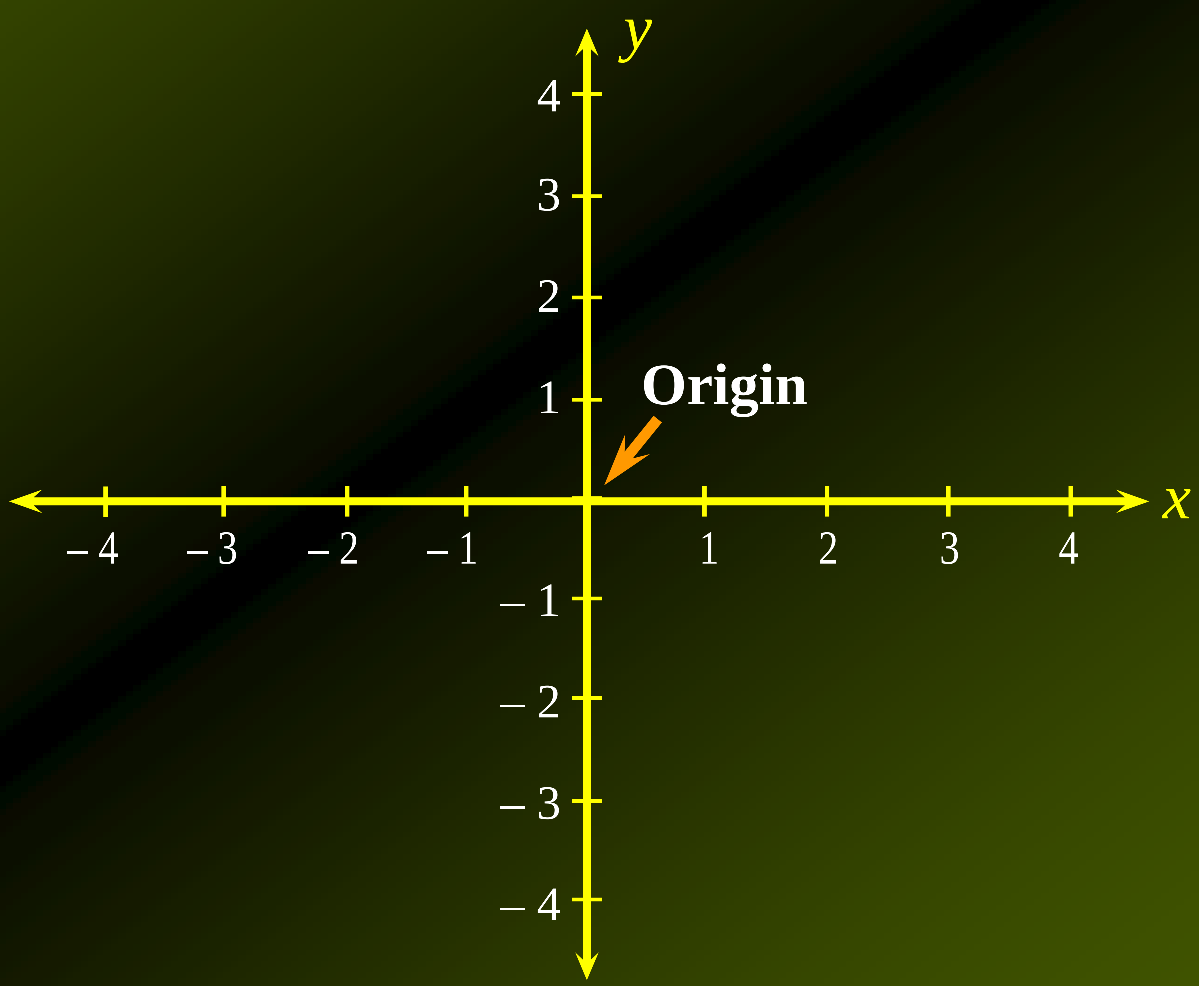
The Cartesian Coordinate System

The horizontal line is called the *x*-axis, and the vertical line is called the *y*-axis.



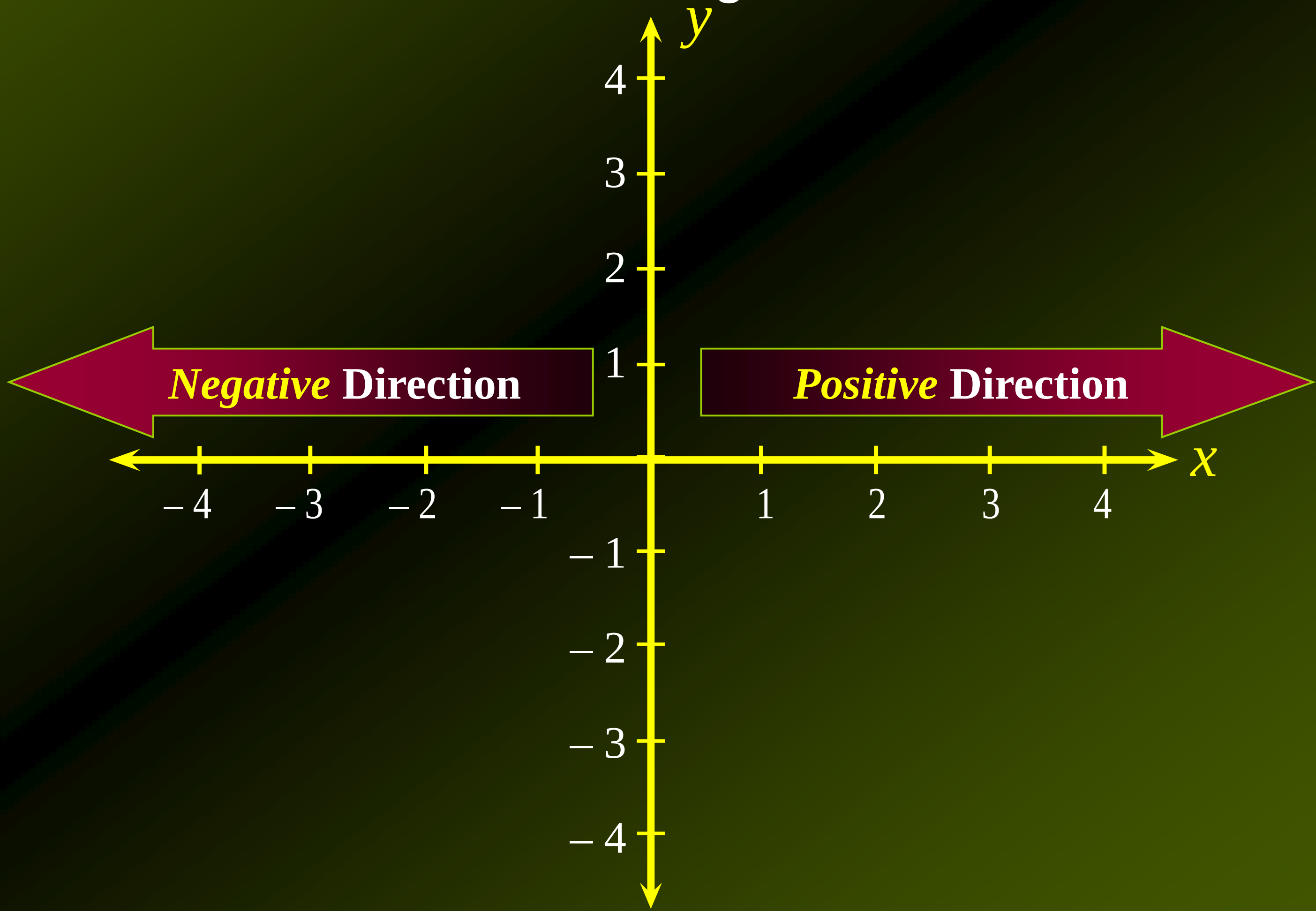
The Cartesian Coordinate System

The point where these two lines intersect is called the **origin**.



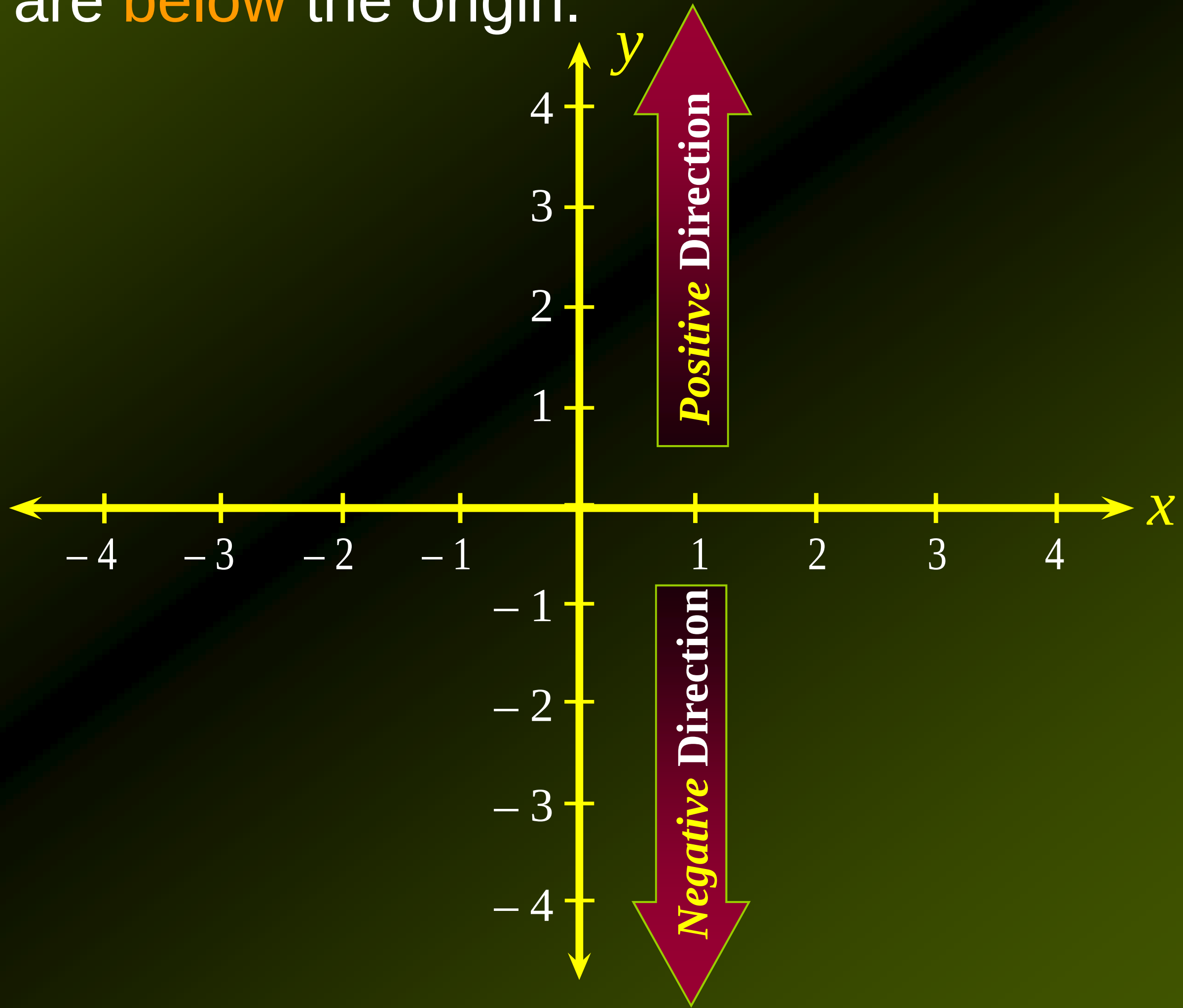
The Cartesian Coordinate System

In the *x*-axis, positive numbers are to the right and negative numbers are to the left of the origin.



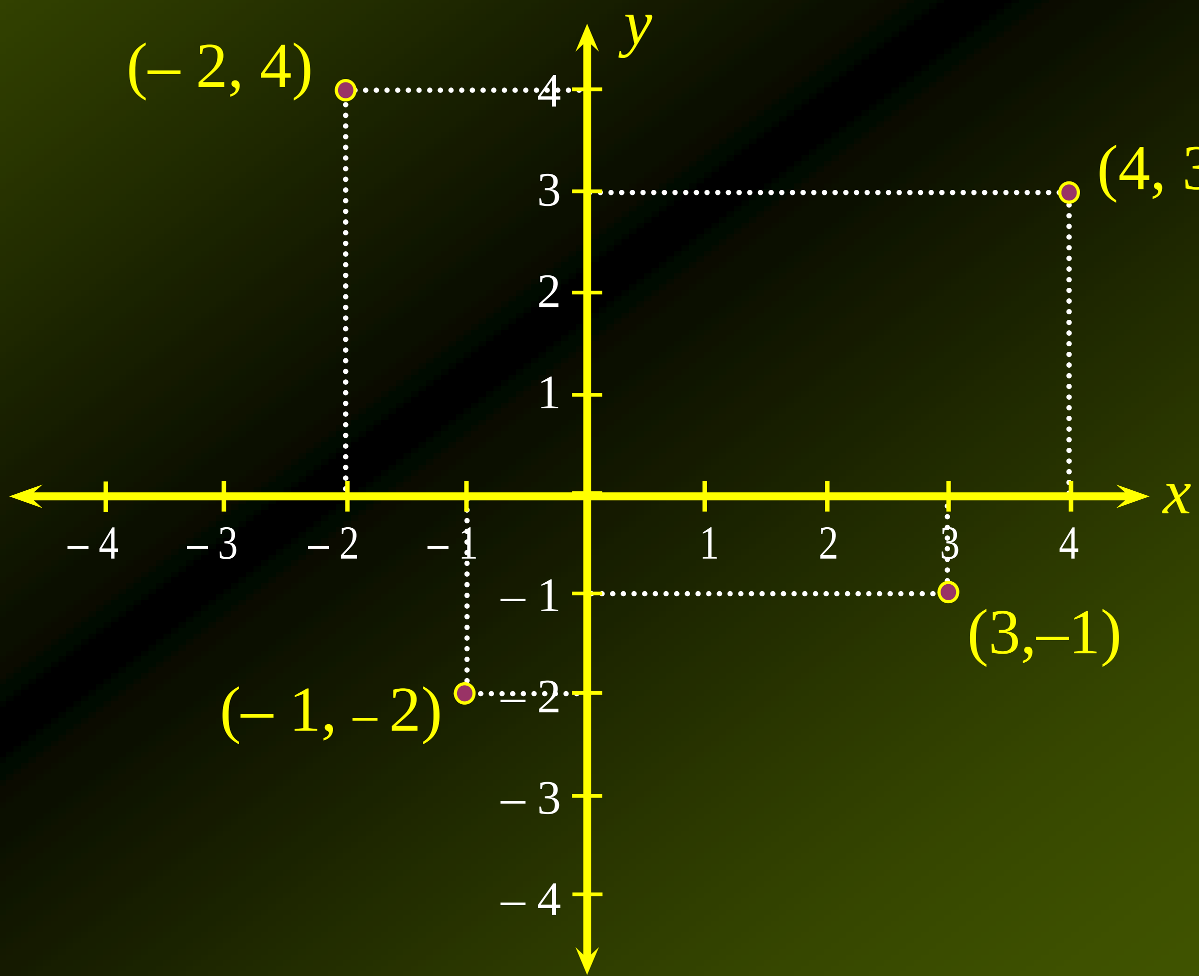
The Cartesian Coordinate System

In the *y*-axis, positive numbers are above and negative numbers are below the origin.



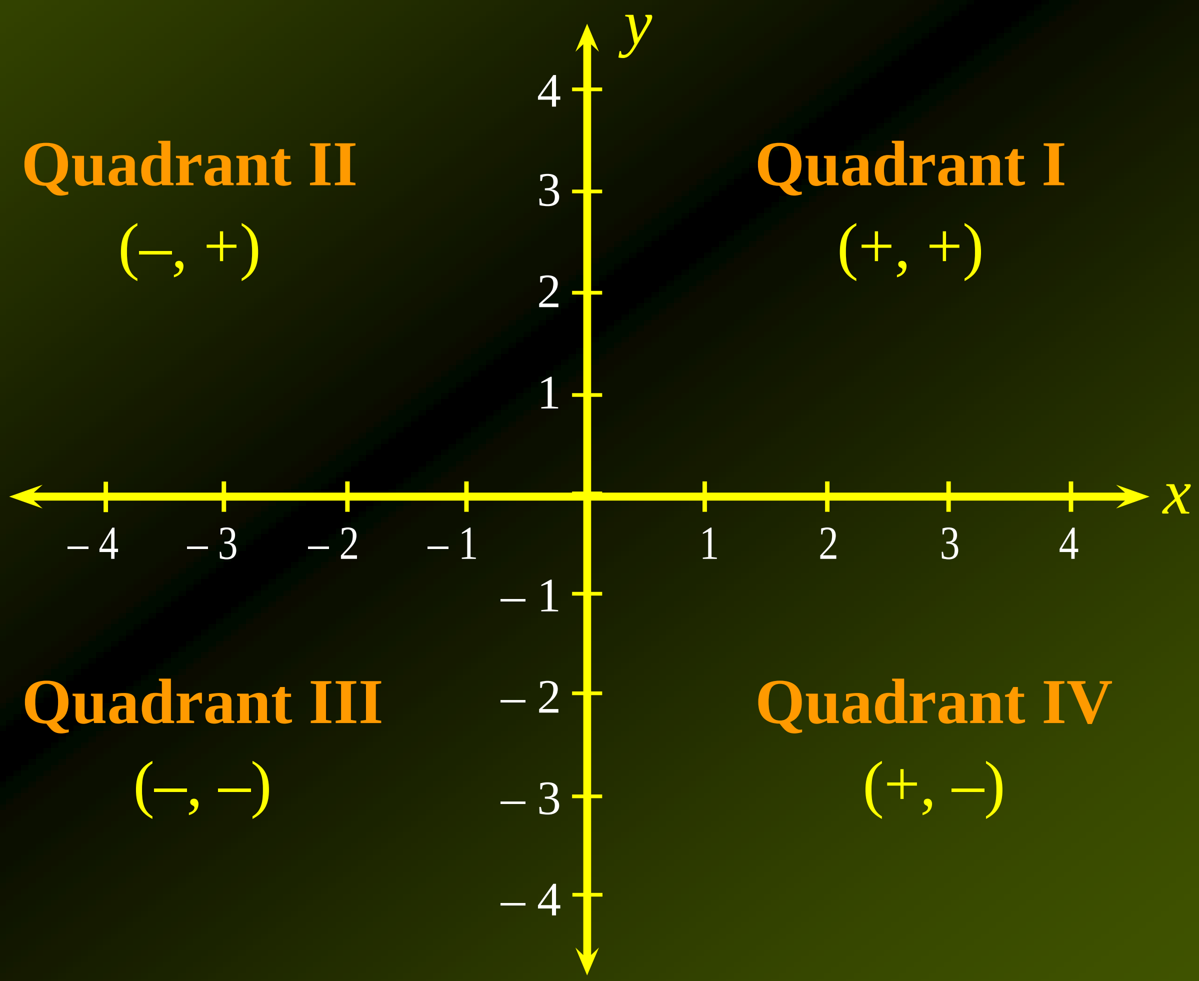
The Cartesian Coordinate System

A point in the plane can now be represented uniquely in this coordinate system by an ordered pair of numbers (x, y).



The Cartesian Coordinate System

The axes divide the plane into four quadrants as shown below.



The Distance Formula

The distance between any two points in the plane may be expressed in terms of their coordinates.

Distance formula

The distance d between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in the plane is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 1

Find the distance between the points (–4, 3) and (2, 6).

Solution:

Let $P_1(-4, 3)$ and $P_2(2, 6)$ be points in the plane.

We have

$x_1 = -4 \qquad y_1 = 3 \qquad x_2 = 2 \qquad y_2 = 6$

Using the distance formula, we have

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{[2-(-4)]^2 + (6-3)^2}$$

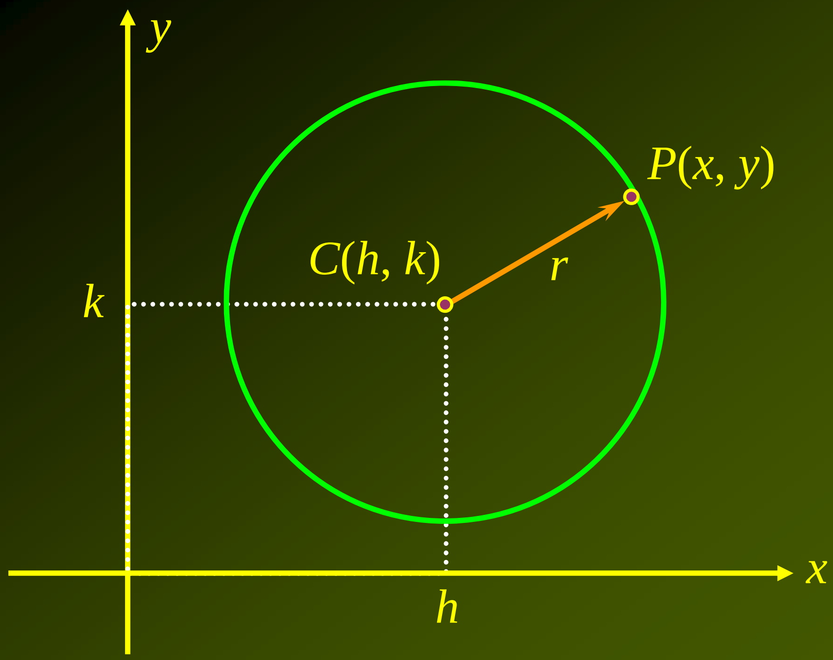
$$= \sqrt{6^2 + 3^2} = \sqrt{45} = 3\sqrt{5}$$

Example 2

Let *P*(*x*, *y*) denote a point lying on the circle with radius *r* and center *C*(*h*, *k*). Find a relationship between *x* and *y*.

Solution:

By definition in a circle, the distance between *P*(*x*, *y*) and *C*(*h*, *k*) is *r*.



With distance formula we get

$$\sqrt{(x-h)^2+(y-k)^2}=r$$

Squaring both sides gives

$$(x-h)^2+(y-k)^2=r^2$$

Equation of a Circle

An equation of a circle with center $C(h, k)$ and radius r is given by

$$(x-h)^2+(y-k)^2=r^2$$

Example 3(a)

Find an equation of the circle with radius 2 and center (–1, 3).

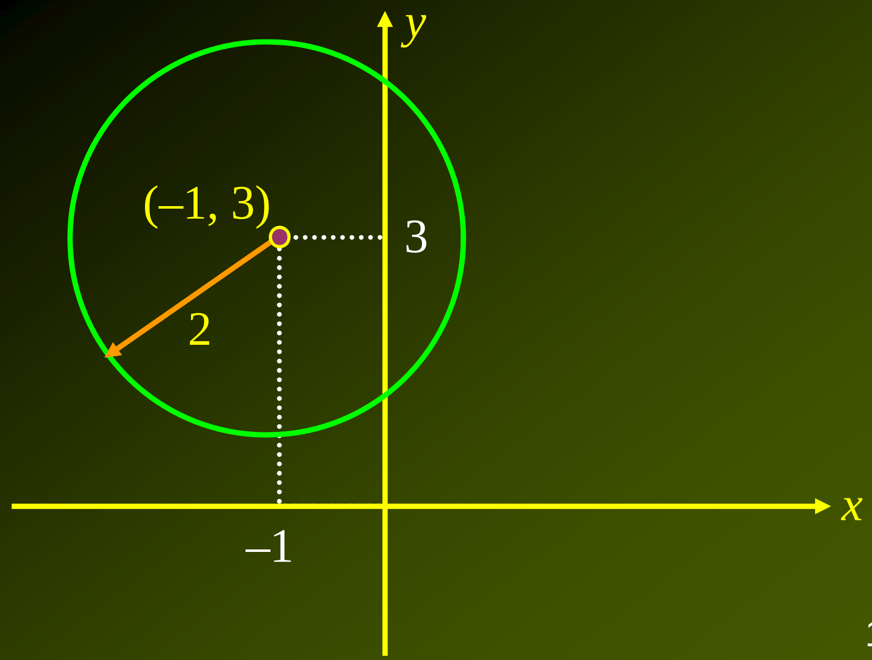
Solution:

We use the circle formula with $r = 2$, $h = -1$, and $k = 3$:

$$(x-h)^2 + (y-k)^2 = r^2$$

$$[x-(-1)]^2 + (y-3)^2 = 2^2$$

$$(x+1)^2 + (y-3)^2 = 4$$



Example 3(b)

Find an equation of the circle with radius 3 and center located at the origin.

Solution:

We use the circle formula with $r = 3$, $h = 0$, and $k = 0$:

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-0)^2 + (y-0)^2 = 3^2$$

$$x^2 + y^2 = 9$$