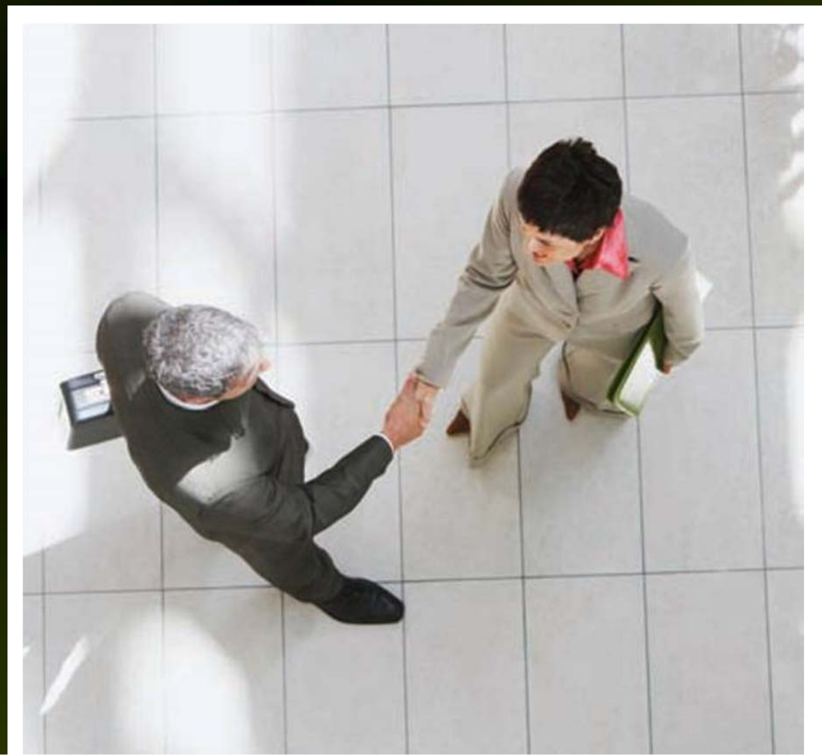
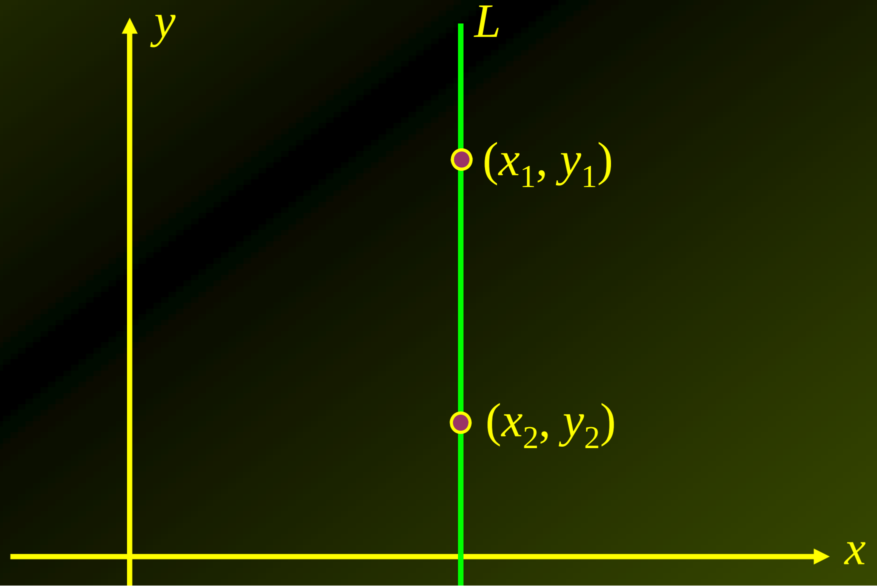
1 PRELIMINARIES



1.4 Straight Lines

ope of a Vertical Line

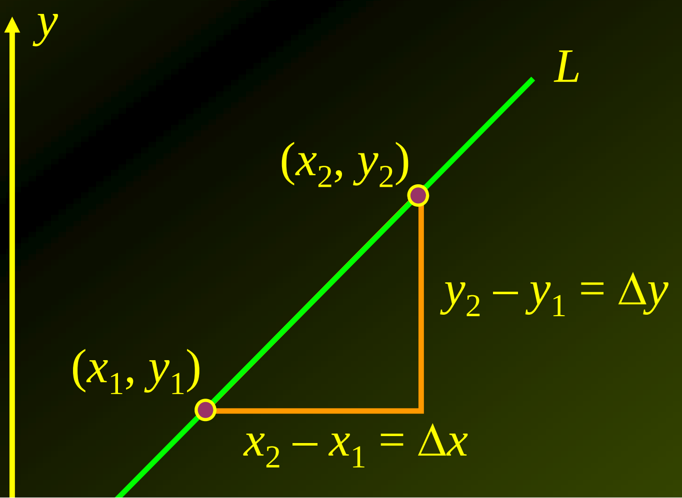
: *L* denote the unique straight line that passes through the
o distinct points (x_1, y_1) and (x_2, y_2). If $x_1 = x_2$, then *L* is a
rtical line, and the slope is undefined.



ope of a Nonvertical Line

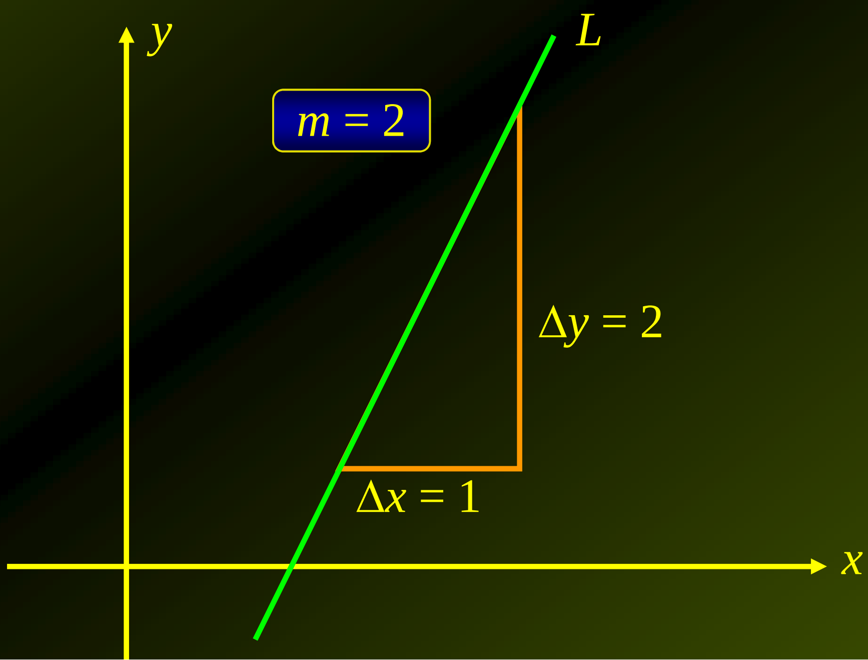
x_1, y_1) and (x_2, y_2) are two distinct points on a nonvertical e L, then the slope m of L is given by

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



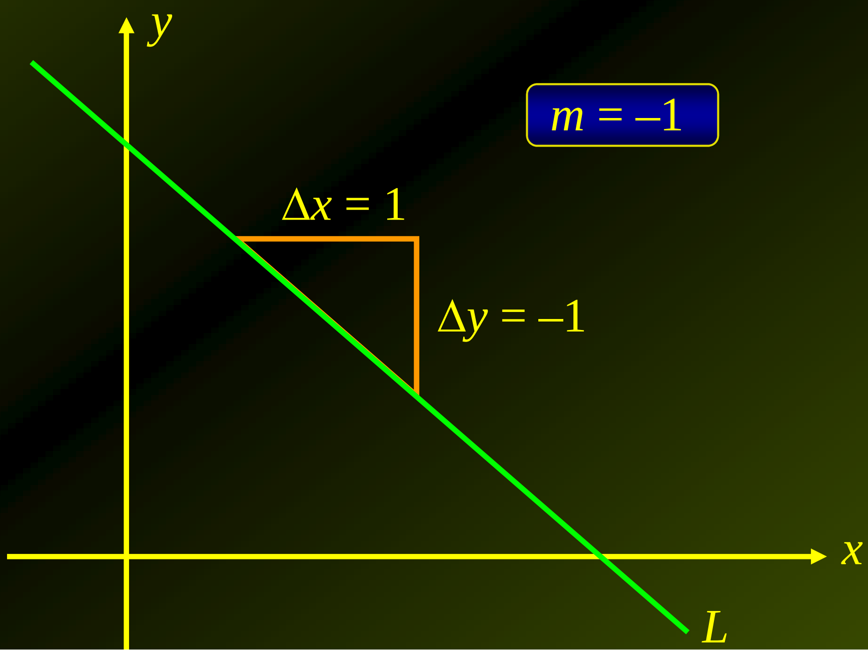
ope of a Nonvertical Line

$m > 0$, the line slants *upward* from left to right.



ope of a Nonvertical Line

n < 0, the line slants *downward* from left to right.



xample 1

etch the straight line that passes through the point
5) and has slope –4/3.

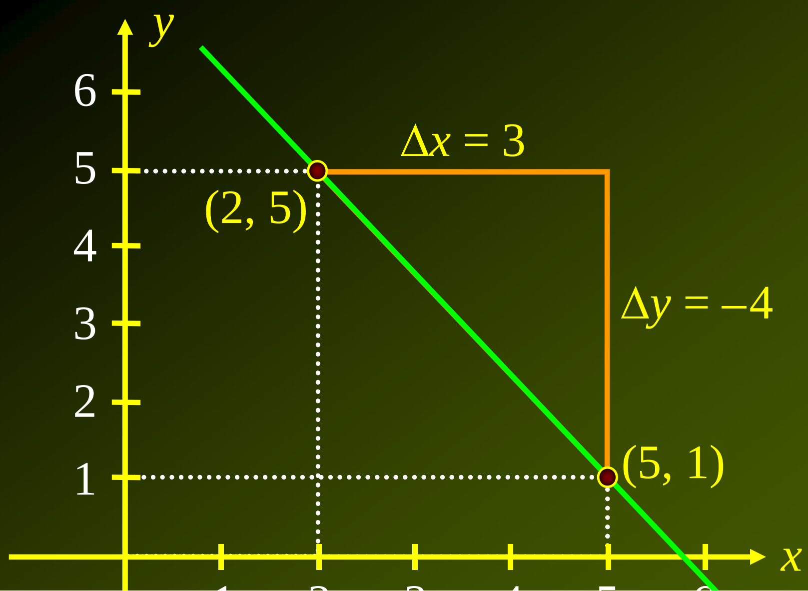
lution:

ot the point (2, 5).

slope of –4/3 means that if
ncreases by 3, *y* decreases
4.

ot the point (5, 1).

aw a line across the two points.



xample 2

nd the slope m of the line that goes through the points
, 1) and (5, 3).

lution:

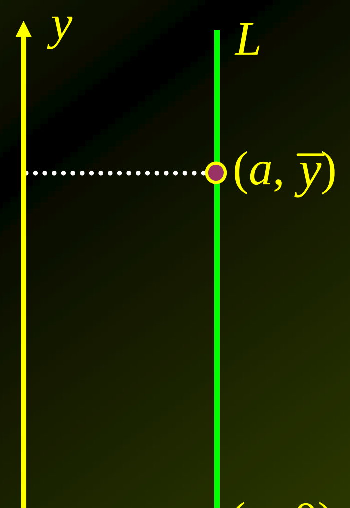
oose (x_1, y_1) to be (–1, 1) and (x_2, y_2) to be (5, 3).

th $x_1 = -1$, $y_1 = 1$, $x_2 = 5$, $y_2 = 3$, we find

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{5 - (-1)} = \frac{2}{6} = \frac{1}{3}$$

quations of Lines

t L be a straight line parallel to the y-axis. Then L crosses e x-axis at some point $(a, 0)$, with the x-coordinate given by = a, where a is a real number. Any other point on L has the m $(a, \bar{y})$, where $\bar{y}$ is an appropriate number. The vertical e L can therefore be described as $x = a$



quations of Lines

t L be a nonvertical line with a slope m.

t (x_1, y_1) be a fixed point lying on L and (x, y) be variable
nt on L distinct from (x_1, y_1).

ing the slope formula by letting $(x, y) = (x_1, y_1)$ we get

$$m = \frac{y - y_1}{x - x_1}$$

ltiplying both sides by $x - x_2$ we get

$$y - y_1 = m(x - x_1)$$

oint-Slope Form

An equation of the line that has slope m and passes through point (x_1, y_1) is given by

$$y - y_1 = m(x - x_1)$$

xample 5

d an equation of the line that passes through the point
3) and has slope 2.

lution:

e the point-slope form

$$y - y_1 = m(x - x_1)$$

bstituting for point (1, 3) and slope $m = 2$, we obtain

$$y - 3 = 2(x - 1)$$

xample 6

d an equation of the line that passes through the points
, 2) and (4, –1).

lution:
e slope is given by

$$m = \frac{y - y_1}{x - x_1} = \frac{-1 - 2}{4 - (-3)} = -\frac{3}{7}$$

bstituting in the point-slope form for point (4, –1) and slope
= –3/7, we obtain

$$y + 1 = -\frac{3}{7}(x - 4)$$

erpendicular Lines

If L_1 and L_2 are two distinct nonvertical lines that have slopes m_1 and m_2, respectively, then L_1 is perpendicular to L_2 (written $L_1 \perp L_2$) if and only if

$$m_1 = -\frac{1}{m_2}$$

xample 7

nd the equation of the line L_1 that passes through the
nt (3, 1) and is perpendicular to the line L_2 described by

$$y - 3 = 2(x - 1)$$

lution:

is described in point-slope form, so its slope is $m_2 = 2$.

nce the lines are perpendicular, the slope of L_1 must be

$$m_1 = -1/2$$

xample 7 – *Solution*

cont'd

ing the point-slope form of the equation for L_1 we obtain

$$y - 1 = -\frac{1}{2}(x - 3)$$

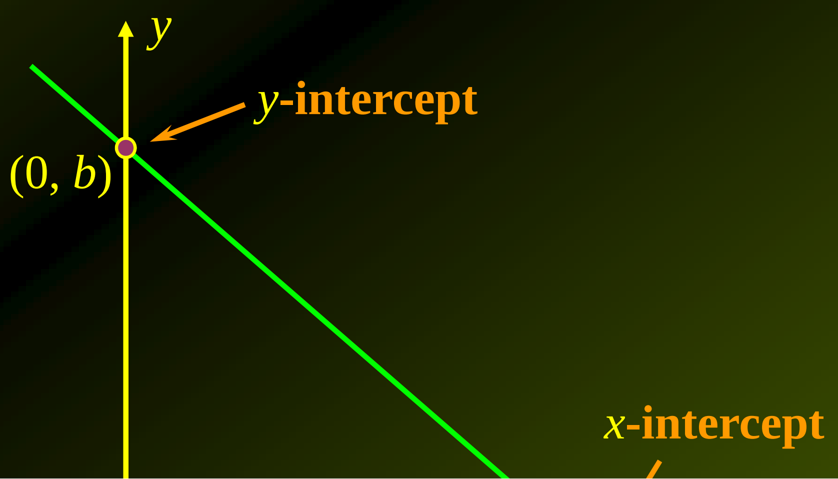
$$2y - 2 = -x + 3$$

$$x + 2y - 5 = 0$$

rossing the Axis

straight line L that is neither horizontal nor vertical cuts the axis and the y-axis at, say, points $(a, 0)$ and $(0, b)$, spectively.

e numbers a and b are called the x-intercept and ntercept, respectively, of L.



ope Intercept Form

An equation of the line that has slope m and intersects the y-axis at the point $(0, b)$ is given by

$$y = mx + b$$

xample 8

nd the equation of the line that has slope 3 and
ntercept of –4.

lution:
e substitute $m = 3$ and $b = -4$ into $y = mx + b$, and get

$$y = 3x - 4$$

xample 9

termine the slope and y-intercept of the line whose
uation is $3x - 4y = 8$.

lution:

write the given equation in the slope-intercept form. Thus,

$$3x - 4y = 8$$

$$-4y = 8 - 3x$$

$$y = \frac{3}{4}x - 2$$

mparing to $y = mx + b$ we find that $m = ¾$ and $b = -2$

plied Example 11

art object purchased for $50,000 is expected to
preciate in value at a constant rate of $5000 per year for
e next 5 years. Write an equation predicting the value of
e art object for any given year. What will be its value 3
ars after the purchase?

pplied Example 11 – *Solution*

:

x = time (in years) since the object was purchased
y = value of object (in dollars)

en, y = 50,000 when x = 0, so the y-intercept is
= 50,000.

ery year the value rises by 5000, so the slope is m = 5000.
us, the equation must be $y = 5000x + 50,000$.

er 3 years the value of the object will be $65,000:

eneral Form of an Linear Equation

The equation

$$Ax + By + C = 0$$

where A, B, and C are constants and A and B are not both zero, is called the general form of a linear equation in the variables x and y.

heorem 1

An equation of a straight line is a linear equation; conversely, every linear equation represents a straight line.

xample 12

etch the straight line represented by the equation

$$3x - 4y - 12 = 0$$

lution:

nce every straight line is uniquely determined by two tinct points, we need find only two such points through ich the line passes in order to sketch it.

r convenience, let's compute the x- and y-intercepts:

- Setting $y = 0$, we find $x = 4$; so the x-intercept is 4.
- Setting $x = 0$, we find $y = -3$; so the y-intercept is -3.

xample 12 – *Solution*

cont'd

aph the line going through the points (4, 0) and (0, –3).