3 DIFFERENTIATION

3.4 Marginal Functions in Economics

arginal Analysis

rginal analysis is the study of the rate of change of
onomic quantities.

ese may have to do with the behavior of costs, revenues,
ofit, output, demand, etc.

this section we will discuss the marginal analysis of
rious functions related to:

- Cost
- Average Cost
- Revenue

plied Example 1 – *Rate of Change of Cost Functions*

ppose the total cost in dollars incurred each week by laraire for manufacturing x refrigerators is given by the al cost function

$$C(x) = 8000 + 200x - 0.2x^2 \qquad (0 \le x \le 400)$$

What is the actual cost incurred for manufacturing the 251st refrigerator?

Find the rate of change of the total cost function with respect to x when $x = 250$.

Compare the results obtained in parts (a) and (b).

pplied Example 1(a) – *Solution*

e cost incurred in producing the 251st refrigerator is

$$C(251) - C(250) = [8000 + 200(251) - 0.2(251)^2] - [8000 + 200(250) - 0.2(250)^2]$$

$$= 45{,}599.8 - 45{,}500$$

$$= 99.80$$

$99.80.

pplied Example 1(b) – *Solution* cont'd

e rate of change of the total cost function

$$C(x) = 8000 + 200x - 0.2x^2$$

h respect to x is given by

$$C'(x) = 200 - 0.4x$$

, when production is 250 refrigerators, the rate of change the total cost with respect to x is

$$C'(x) = 200 - 0.4(250)$$

$$= 100$$

$100.

pplied Example 1(c) – *Solution*

cont'd

mparing the results from (a) and (b) we can see they are ry similar: \$99.80 versus \$100.

is is because (a) measures the average rate of change er the interval [250, 251], while (b) measures the tantaneous rate of change at exactly $x = 250$.

e *smaller* the interval used, the *closer* the average rate change becomes to the instantaneous rate of change.

arginal Analysis

e actual cost incurred in producing an additional unit of a
od is called the marginal cost.

we just saw, the marginal cost is approximated by the
e of change of the total cost function.

r this reason, economists define the marginal cost
nction as the derivative of the total cost function.

plied Example 2 – *Marginal Cost Functions*

subsidiary of Elektra Electronics manufactures a portable
usic player. Management determined that the daily total
st of producing these players (in dollars) is

$$C(x) = 0.0001x^3 - 0.08x^2 + 40x + 5000$$

ere x stands for the number of players produced.

Find the marginal cost function.

Find the marginal cost for x = 200, 300, 400, and 600.

nterpret your results.

pplied Example 2(a) – *Solution*

he total cost function is:

$$C(x) = 0.0001x^3 - 0.08x^2 + 40x + 5000$$

en, its derivative is the marginal cost function:

$$C'(x) = 0.0003x^2 - 0.16x + 40$$

pplied Example 2(b) – *Solution* cont'd

e marginal cost for x = 200, 300, 400, and 600 is:

$$C'(200) = 0.0003(200)^2 - 0.16(200) + 40 = 20$$

$$C'(300) = 0.0003(300)^2 - 0.16(300) + 40 = 19$$

$$C'(400) = 0.0003(400)^2 - 0.16(400) + 40 = 24$$

$$C'(600) = 0.0003(600)^2 - 0.16(600) + 40 = 52$$

$20/unit, $19/unit, $24/unit, and $52/unit, respectively.

pplied Example 2(c) – *Solution*

cont'd

om part (b) we learn that *at first* the marginal cost is creasing, but *as output increases*, the marginal cost reases as well.

is is a common phenomenon that occurs because of veral factors, such as excessive costs due to overtime d high maintenance costs for keeping the plant running such a fast rate.

plied Example 5 – *Marginal Revenue Functions*

ppose the relationship between the unit price p in dollars d the quantity demanded x of the Acrosonic model F dspeaker system is given by the equation

$$p = -0.02x + 400 \qquad (0 \le x \le 20{,}000)$$

Find the revenue function R.

Find the marginal revenue function R'.

Compute $R'(2000)$ and interpret your result.

pplied Example 5(a) – *Solution*

e revenue function is given by

$$R(x) = px$$

$$= (–0.02x + 400)x$$

$$= –0.02x^2 + 400x \qquad (0 \le x \le 20{,}000)$$

pplied Example 5(b) – *Solution*

cont’d

ven the revenue function

$$R(x) = –0.02x^2 + 400x$$

e find its derivative to obtain the marginal revenue
nction:

$$R'(x) = –0.04x + 400$$

pplied Example 5(c) – *Solution* cont'd

hen quantity demanded is 2000, the marginal revenue
l be:

$$R'(2000) = -0.04(2000) + 400$$

$$= 320$$

us, the actual revenue realized from the sale of the
01st loudspeaker system is approximately $320.

plied Example 6 – *Marginal Profit Function*

ntinuing with the last example, suppose the total cost (in lars) of producing x units of the Acrosonic model F dspeaker system is

$$C(x) = 100x + 200{,}000$$

Find the profit function P.

Find the marginal profit function P'.

Compute $P'(2000)$ and interpret the result.

plied Example 6(a) – *Solution*

om last example we know that the revenue function is

$$R(x) = -0.02x^2 + 400x$$

ofit is the difference between total revenue and total cost, the profit function is

$$P(x) = R(x) - C(x)$$

$$= (-0.02x^2 + 400x) - (100x + 200{,}000)$$

$$= -0.02x^2 + 300x - 200{,}000$$

pplied Example 6(b) – *Solution*

cont'd

ven the profit function

$$P(x) = -0.02x^2 + 300x - 200{,}000$$

find its derivative to obtain the marginal profit function:

$$P'(x) = -0.04x + 300$$

pplied Example 6(c) – *Solution* cont'd

hen producing $x = 2000$, the marginal profit is

$$P'(2000) = -0.04(2000) + 300$$

$$= 220$$

us, the profit to be made from producing the 2001st dspeaker is \$220.

asticity of Demand

onomists are frequently concerned with *how strongly* do anges in prices *cause* quantity demanded to change.

e measure of the strength of this reaction is called the sticity of demand, which is given by

$$E(p) = -\frac{\text{percentage change in quantity demanded}}{\text{percentage change in price}}$$

te:

nce the ratio is negative, economists use the negative of e ratio, to make the elasticity be a positive number.

asticity of Demand

ppose the price of a good increases by *h* dollars from *p*
(*p* + *h*) dollars.

e percentage change of the price is

$$\text{Percentage change in price} = \frac{\text{Change in price}}{\text{Price}}(100) = \frac{h}{p}(100)$$

e percentage change in quantity demanded is

$$\text{Percentage change in quantity demanded} = \frac{\text{Change in quantity demanded}}{\text{Quantity demanded at price } p}(100)$$

asticity of Demand

e good way to measure the effect that a percentage ange in price has on the percentage change in the antity demanded is to look at the ratio of the latter to the mer. We find

$$E(p) = \frac{\text{Percentage change in quantity demanded}}{\text{Percentage change in price}} = \frac{\left[\dfrac{f(p+h)-f(p)}{f(p)}\right](100)}{\left(\dfrac{h}{p}\right)(100)}$$

$$= \frac{\dfrac{f(p+h)-f(p)}{f(p)}}{\cdots} \qquad = \frac{\dfrac{f(p+h)-f(p)}{h}}{\cdots}$$

asticity of Demand

e have

$$E(p) = \frac{\frac{f(p+h) - f(p)}{h}}{\frac{f(p)}{p}}$$

is differentiable at *p*, then, when ***h*** is small,

$$\frac{f(p+h) - f(p)}{h} \approx f'(p)$$

erefore, if ***h*** is small, the ratio is approximately equal to

$$E(p) = \frac{f'(p)}{\frac{f(p)}{p}} = \frac{pf'(p)}{f(p)}$$

asticity of Demand

Elasticity of Demand

If *f* is a differentiable demand function defined by $x = f(p)$, then the elasticity of demand at price *p* is given by

$$E(p) = -\frac{pf'(p)}{f(p)}$$

te:

nce the ratio is negative, economists use the negative of

plied Example 7 – *Elasticity of Demand*

nsider the demand equation for the Acrosonic model F dspeaker system:

$$p = -0.02x + 400 \qquad (0 \le x \le 20{,}000)$$

Find the elasticity of demand $E(p)$.

Compute $E(100)$ and interpret your result.

Compute $E(300)$ and interpret your result.

pplied Example 7(a) – *Solution*

lving the demand equation for x in terms of p, we get

$$x = f(p) = -50p + 20{,}000$$

om which we see that

$$f'(p) = -50$$

erefore,

$$E(p) = -\frac{pf'(p)}{f(p)}$$

$$= -\frac{p(-50)}{-50p + 20{,}000}$$

pplied Example 7(b) – *Solution*

cont'd

hen $p = 100$ the elasticity of demand is

$$E(100) = \frac{(100)}{400-(100)} = \frac{1}{3}$$

is means that for every 1% increase in price we can pect to see a 1/3% decrease in quantity demanded.

cause the response (change in quantity demanded) is *s* than the action (change in price), we say demand is *lastic*.

plied Example 7(c) – *Solution*

cont'd

hen $p = 300$ the elasticity of demand is

$$E(300) = \frac{(300)}{400-(300)} = 3$$

is means that for every 1% increase in price we can
pect to see a 3% decrease in quantity demanded.

cause the response (change in quantity demanded) is
eater than the action (change in price), we say demand is
stic.

mand is said to be *elastic* whenever $E(p) > 1$.