3 DIFFERENTIATION

3.5 Higher Order Derivatives

gher-Order Derivatives

e derivative f' of a function f is also a function. As such, may also be differentiated.

us, the function f' has a derivative f'' at a point x in the main of f if the limit of the quotient

$$\frac{f'(x+h) - f'(x)}{h}$$

sts as h approaches zero. The function f'' obtained in s manner is called the second derivative of the function f, t as the derivative f' of f is often called the first derivative f.

xample 1

nd the third derivative of the function $f(x) = x^{2/3}$ and termine its domain.

lution:

e have $f'(x) = \frac{2}{3}x^{-1/3}$ and $f''(x) = \frac{2}{3}\left(-\frac{1}{3}\right)x^{-4/3} = -\frac{2}{9}x^{-4/3}$

the required derivative is

$$f'''(x) = -\frac{2}{9}\left(-\frac{4}{3}\right)x^{-7/3} = \frac{8}{27}x^{-7/3} = \frac{8}{27x^{7/3}}$$

e domain of the third derivative is the set of all real

xample 2(a)

nd the second derivative of the function $f(x) = (2x^2 + 3)^{3/2}$

lution:

sing the general power rule we get the first derivative:

$$f'(x) = \frac{3}{2}\left(2x^2 + 3\right)^{1/2}(4x)$$

$$= 6x\left(2x^2 + 3\right)^{1/2}$$

xample 2(b)

nd the second derivative of the function $f(x) = (2x^2 + 3)^{3/2}$

lution:

sing the product rule we get the second derivative:

$$f''(x) = 6x \cdot \frac{d}{dx}\left(2x^2+3\right)^{1/2} + \left(2x^2+3\right)^{1/2} \cdot \frac{d}{dx}(6x)$$

$$= 6x \cdot \left(\frac{1}{2}\right)\left(2x^2+3\right)^{-1/2}(4x) + \left(2x^2+3\right)^{1/2} \cdot 6$$

$$= 12x^2\left(2x^2+3\right)^{-1/2} + 6\left(2x^2+3\right)^{1/2}$$

$$= 6\left(2x^2+3\right)^{-1/2}\left[2x^2 + \left(2x^2+3\right)\right]$$

plied Example 4 – *Acceleration of a Maglev*

e distance s (in feet) covered by a maglev moving along traight track t seconds after starting from rest is given by function

$$s = 4t^2 \qquad (0 \leq t \leq 10)$$

hat is the maglev's acceleration after 30 seconds?

lution:

e velocity of the maglev t seconds from rest is given by

$$v = \frac{ds}{dt}$$

$$\frac{d}{dt}(4t^2) = 8t$$

pplied Example 4 – *Solution*

cont'd

e acceleration of the maglev *t* seconds from rest is given the rate of change of the velocity of *t*, given by

$$a = \frac{d}{dt}v$$

$$= \frac{d}{dt}\left(\frac{ds}{dt}\right)$$

$$= \frac{d^2s}{dt^2}$$

$$= \frac{d}{dt}(8t) = 8$$