5 EXPONENTIAL AND LOGARITHMIC FUNCTIONS

5.4 Differentiation of the Exponential Function

Rule 1: Derivative of the Exponential Function

The derivative of the exponential function with base *e* is equal to the function itself:

$$\frac{d}{dx}\left(e^x\right) = e^x$$

Example 1(a)

Find the derivative of the function $f(x) = x^2 e^x$

Solution:

Using the product rule gives

$$f'(x) = \frac{d}{dx}\left(x^2 e^x\right)$$

$$= x^2 \frac{d}{dx}\left(e^x\right) + e^x \frac{d}{dx}\left(x^2\right)$$

$$= x^2 e^x + e^x (2x)$$

$$= xe^x (x+2)$$

Example 1(b)

Find the derivative of the function $g(t) = \left(e^t + 2\right)^{3/2}$

Solution:

Using the general power rule gives

$$g'(t) = \frac{3}{2}\left(e^t + 2\right)^{1/2} \frac{d}{dt}\left(e^t + 2\right)$$

$$= \frac{3}{2}\left(e^t + 2\right)^{1/2} e^t$$

$$= \frac{3}{2} e^t \left(e^t + 2\right)^{1/2}$$

Rule 2: Chain Rule for Exponential Functions

If $f(x)$ is a differentiable function, then

$$\frac{d}{dx}\left(e^{f(x)}\right) = e^{f(x)} f'(x)$$

Example 2(a)

Find the derivative of the function $f(x) = e^{2x}$

Solution:

$$f'(x) = e^{2x} \frac{d}{dx}(2x)$$

$$= e^{2x}(2)$$

$$= 2e^{2x}$$

Example 2(b)

Find the derivative of the function $y = e^{-3x}$

Solution:

$$\frac{dy}{dx} = e^{-3x} \frac{d}{dx}(-3x)$$

$$= e^{-3x}(-3)$$

$$= -3e^{-3x}$$

Example 2(c)

Find the derivative of the function $g(t) = e^{2t^2 + t}$

Solution:

$$g'(t) = e^{2t^2+t} \cdot \frac{d}{dt}\left(2t^2 + t\right)$$

$$= (4t+1)e^{2t^2+t}$$

Example 3

Find the derivative of the function $y = xe^{-2x}$

Solution:

$$\frac{dy}{dx} = x\frac{d}{dx}\left(e^{-2x}\right) + e^{-2x}\frac{d}{dx}(x)$$

$$= x\left[e^{-2x}\frac{d}{dx}(-2x)\right] + e^{-2x}(1)$$

$$= xe^{-2x}(-2) + e^{-2x}$$

$$= -2xe^{-2x} + e^{-2x}$$

$$= e^{-2x}(1-2x)$$

Example 4

Find the derivative of the function $g(t)=\dfrac{e^t}{e^t+e^{-t}}$

Solution:

$$g'(t)=\frac{\left(e^t+e^{-t}\right)\dfrac{d}{dt}\left(e^t\right)-e^t\dfrac{d}{dt}\left(e^t+e^{-t}\right)}{\left(e^t+e^{-t}\right)^2}$$

$$=\frac{\left(e^t+e^{-t}\right)e^t-e^t\left(e^t-e^{-t}\right)}{\left(e^t+e^{-t}\right)^2}$$

$$=\frac{e^{2t}+1-e^{2t}+1}{\left(e^t+e^{-t}\right)^2}$$

$$=\frac{2}{\left(e^t+e^{-t}\right)^2}$$

Example 6

Find the inflection points of the function $f(x) = e^{-x^2}$

Solution:

Find the first and second derivatives of *f* :

$$f'(x) = -2xe^{-x^2}$$

$$f''(x) = (-2x)(-2xe^{-x^2}) - 2e^{-x^2}$$

$$= 2e^{-x^2}(2x^2 - 1)$$

Setting $f'' = 0$ gives $e^{-x^2} = 0$, and $2x^2 - 1 = 0$.

Example 6 – *Solution*

cont'd

Since e^{-x^2} never equals zero for any real value of x, the only candidates for inflection points of f are

$$x = \pm 1/\sqrt{2}$$

Testing values around these numbers we conclude that they are indeed inflection points.