6 INTEGRATION

6.3 Area and the Definite Integral

The Area Under the Graph of a Function

Let *f* be a nonnegative continuous function on [*a*, *b*]. Then, the area of the region under the graph of *f* is

$$A = \lim_{n \to \infty} \left[f(x_1) + f(x_2) + \ldots + f(x_n) \right] \Delta x$$

where $x_1, x_2, \ldots, x_n$ are arbitrary points in the *n* subintervals of [*a*, *b*] of equal width $\Delta x = (b - a)/n$.

The Definite Integral

Let f be a continuous function defined on $[a, b]$. If

$$\lim_{n\to\infty}\left[f(x_1)\Delta x + f(x_2)\Delta x + \ldots + f(x_n)\Delta x\right]$$

exists for all choices of representative points $x_1, x_2, \ldots , x_n$ in the n subintervals of $[a, b]$ of equal width $\Delta x = (b - a)/n$, then the limit is called the definite integral of f from a to b and is denoted by

$$\int_a^b f(x)dx$$

Thus,

$$\int_a^b f(x)dx = \lim_{n\to\infty}\left[f(x_1)\Delta x + f(x_2)\Delta x + \ldots + f(x_n)\Delta x\right]$$

The number a is the lower limit of integration, and the number b is the upper limit of integration.

Integrability of a Function

Let *f* be a continuous on [*a*, *b*]. Then, *f* is integrable on [*a*, *b*]; that is, the definite integral

$$\int_a^b f(x)dx$$

exists.

Geometric Interpretation of the Definite Integral

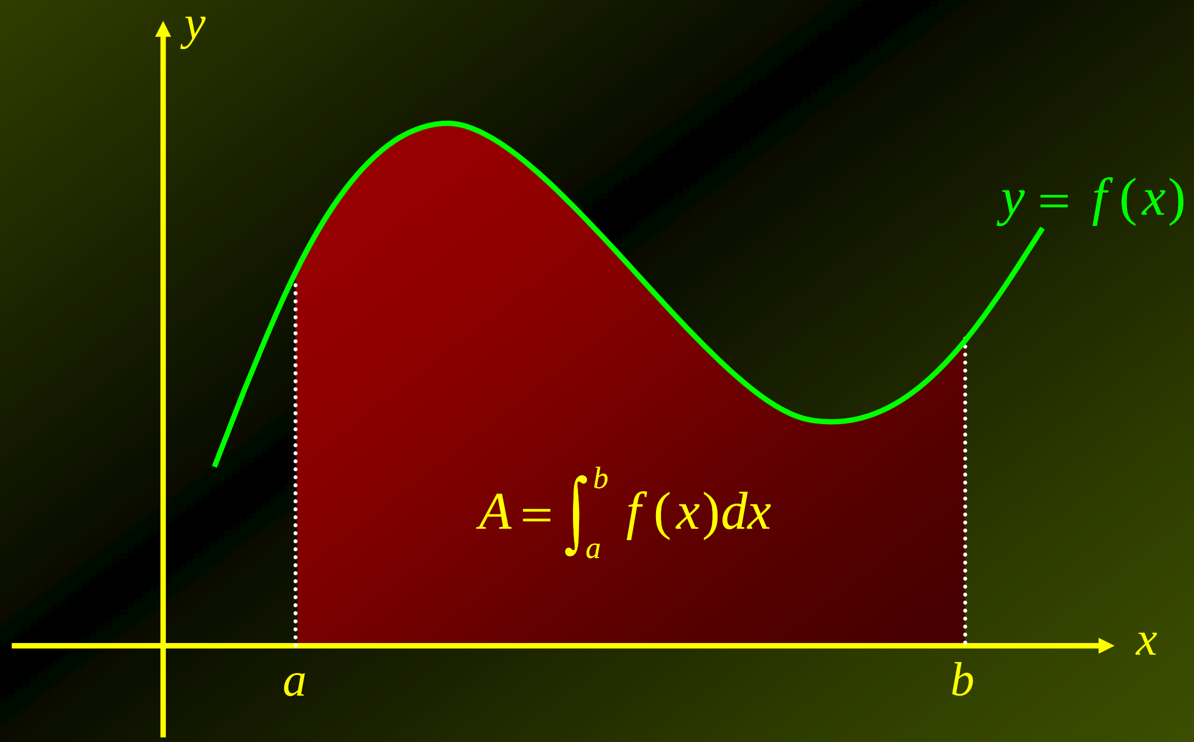
If f is nonnegative and integrable on $[a, b]$, then

$$\int_a^b f(x)dx$$

is equal to the area of the region under the graph of f on $[a, b]$.

Geometric Interpretation of the Definite Integral

The definite integral is equal to the area of the region under the graph of *f* on [*a*, *b*]:



Geometric Interpretation of the Definite Integral

If *f* is continuous on [*a*, *b*], then

$$\int_a^b f(x)dx$$

is equal to the area of the region above [*a*, *b*] minus the region below [*a*, *b*].

Geometric Interpretation of the Definite Integral

The definite integral is equal to the area of the region above [*a*, *b*] minus the region below [*a*, *b*]:

$$A = \int_a^b f(x)dx = R_1 - R_2 + R_3$$