7 ADDITIONAL TOPICS IN INTEGRATION

7.2 Integration Using Tables of Integrals

A Table of Integrals

We have covered several techniques for finding the antiderivatives of functions. There are many more such techniques and extensive integration formulas have been developed for them.

You can find a table of integrals on pages 491 and 492 of the text that include some such formulas for your benefit. We will now consider some examples that illustrate how this table can be used to evaluate an integral.

Example 1

Use the table of integrals to find $\int \frac{2x\,dx}{\sqrt{3+x}}$

Solution:

We first rewrite $\int \frac{2x\,dx}{\sqrt{3+x}} = 2\int \frac{x\,dx}{\sqrt{3+x}}$

Since $\sqrt{3+x}$ is of the form $\sqrt{a+bu}$, with $a = 3$, $b = 1$, and $u = x$, we use Formula (5),

$$\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu} + C$$

obtaining

$$2\int \frac{x\,dx}{\sqrt{3+x}} = 2\left[\frac{2}{3(1)^2}(x-2\cdot 3)\sqrt{3+x}\right] + C = \frac{4}{3}(x-6)\sqrt{3+x} + C$$

Example 2

Use the table of integrals to find $\int x^2\sqrt{3+x^2}\,dx$

Solution:

We first rewrite 3 as $(\sqrt{3})^2$, so that $\sqrt{3+x^2}$ has the form $\sqrt{a^2+u^2}$ with $a=\sqrt{3}$ and $u = x$.

Using Formula (8),

$$\int u^2\sqrt{a^2+u^2}\,du = \frac{u}{8}(a^2+2u^2)\sqrt{a^2+u^2} - \frac{a^4}{8}\ln\left|u+\sqrt{a^2+u^2}\right| + C$$

obtaining

$$\int x^2\sqrt{3+x^2}\,dx = \frac{x}{8}(3+2x^2)\sqrt{3+x^2} - \frac{9}{8}\ln\left|x+\sqrt{3+x^2}\right| + C$$

Example 5(a)

Use the table of integrals to find $\int x^2 e^{(-1/2)x} dx$

Solution:

We can use Formula (24),

$$\int u^n e^{au} du = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} du$$

Letting $n = 2$, $a = -½$, and $u = x$, we have

$$\int x^2 e^{(-1/2)x} dx = \frac{1}{(-\frac{1}{2})} x^2 e^{(-1/2)x} - \frac{2}{(-\frac{1}{2})} \int x e^{(-1/2)x} dx$$

$$= -2x^2 e^{(-1/2)x} + 4\int x e^{(-1/2)x} dx$$

Example 5(b)

Use the table of integrals to find $\int x^2 e^{(1/2)x} dx$

Solution:

We have $\int x^2 e^{(1/2)x} dx = -2x^2 e^{(-1/2)x} + 4\int xe^{(-1/2)x} dx$

Using Formula (24) again, with n = 1, a = – ½, and u = x, we get

$$\int x^2 e^{(-1/2)x} dx = -2x^2 e^{(-1/2)x} + 4\left[\frac{1}{(-\frac{1}{2})} xe^{(-1/2)x} - \frac{1}{(-\frac{1}{2})}\int e^{(-1/2)x} dx\right]$$

$$= -2x^2 e^{(-1/2)x} + 8\left[-xe^{(-1/2)x} + \int e^{(-1/2)x} dx\right]$$

Example 5(b) – *Solution*

cont'd

$$= -2x^2e^{(-1/2)x} + 8\left[-xe^{(-1/2)x} + \frac{1}{(-\frac{1}{2})}e^{(-1/2)x}\right] + C$$

$$= -2e^{(-1/2)x}(x^2 + 4x + 8) + C$$

Applied Example 6 – *Mortgage Rates*

A study prepared for the National Association of realtors estimated that the mortgage rate over the next *t* months will be

$$r(t) = \frac{6t + 75}{t + 10} \qquad (0 \le t \le 24)$$

percent per year. If the prediction holds true, what will be the average mortgage rate over the 12 months?

Applied Example 6 – *Solution*

The average mortgage rate over the next 12 months will be given by

$$A = \frac{1}{12-0}\int_0^{12} \frac{6t+75}{t+10}\,dt$$

$$= \frac{1}{12}\left[\int_0^{12} \frac{6t}{t+10}\,dt + \int_0^{12} \frac{75}{t+10}\,dt\right]$$

$$= \frac{1}{2}\int_0^{12} \frac{t}{t+10}\,dt + \frac{25}{4}\int_0^{12} \frac{1}{t+10}\,dt$$

Applied Example 6 – *Solution*

cont'd

Use Formula (1)

$$\int \frac{udu}{a+bu} = \frac{1}{b^2}[a+bu-a\ln|a+bu|]+C$$

to evaluate the first integral

$$A = \frac{1}{2}[10+t-10\ln(10+t)]\Big|_0^{12} + \frac{25}{4}\ln(10+t)\Big|_0^{12}$$

$$= \frac{1}{2}[(22-10\ln 22)-(10-10\ln 10)] + \frac{25}{4}[\ln 22 - \ln 10]$$

$$\approx 6.99$$

or approximately 6.99% per year.